Method of Moment Determination of Current Distribution on Elements of Yagi-Uda Array

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ABSTRACT
Presented in this paper is the numerical solution to the current distributions on two forms of Yagi-Uda antenna designs. One form consists of twelve elements while the other consists of fourteen elements. Employing method of moments technique in which the unknown current is expanded in terms of known expansion function and complex coefficients which are to be determined. It is demonstrated that, when the integral equation that expresses tangential component of an impressed field in terms of induced current on the elements of Yagi-Uda array is reduced into matrix form, the current distribution of interest becomes known. The profiles for the current distributions on elements of those arrays represented in graphical forms reveal that, the currents are symmetrical about the length of the element in each case. It is found that the highest magnitude of the current exists on the driven element. Furthermore, the characteristic profiles of the currents on elements of those arrays exhibit sinusoidal type of waveform and are largely similar when the frequencies of operation are 200MHz and 665MHz, respectively.

Keywords
Current distribution, Method of moments, Wire antenna, Yagi-Uda antenna

1. INTRODUCTION
Pocklington in 1897 explained how thin wire antennas radiated electromagnetic energy in free space using Hertz’z potential theory which was deemed to be the first analytical investigation of problem of wire antenna [1]. Ever since, the analysis of wire antennas of different geometrical shapes has attracted interests from number of researchers whose various contributions have provided useful information about their radiation characteristics. For instance, numerical results are computed for electric field radiated and radiation resistance of wire antenna of circular zig-zag shape [2]. The cited work utilized the approximation of uniform current distribution and it was shown that, the directivity of field pattern could be adjusted by varying the number of zig zag sections. Using an assumption of sinusoidal current distribution, radiation resistance of wire antenna of circular shape of larger circumference is determined [3]. The paper under reference here posits that, sinusoidal current distribution is an accurate assumption for circular loop of larger sizes. A particular wire antenna which has been engaged as roof top television receiver and transmission amongst other uses is Yagi-Uda antenna. The simplest form of the antenna consists of a reflector, driven element and a director with each element performing different role. The reflector acts to suppress the electromagnetic radiation in undesired backward direction, the director on other hand focuses the electromagnetic wave in the preferred direction. The driven element provides means by which the antenna is excited.

Representative research efforts on the antenna include the determination of the radiation characteristics of four-element and six-element Yagi-Uda antennas using simulation software tools [4], [5]. The design parameters of the antenna have been optimized to evolve Yagi-antenna of satisfactory performance [6]. Utilizing an assumption of sinusoidal current distribution as an approximate excitation current on the antenna, the gain characteristics, the near field as well as the far fields of the antenna are determined [7], [8]. However, that restrictive assumption has been disputed in the literature [9], with the latter adopting the use of three term’s theory proposed by Professor King and his companions to increase the gain of Yagi-Uda antenna. The picture painted in the foregoing raises questions about the nature of the current distribution on Yagi-Uda array elements. This research work therefore revisits the problem using generalized technique, popularly known as Method of Moments to solve for the current distribution on elements of two models of Yagi-Uda array, one consists of twelve elements and the other comprises fourteen elements. The paper is divided into various sections such that section I focuses on introduction into the subject matter, while section II concerns itself with provision of background information about method of moment technique. Section III utilizes the method to determine the current distribution on elements of Yagi-Uda array which is discussed in section IV. Section V presents the summary of the entire work.

2. THEORY
The advent of high speed digital computers introduces a number of numerical techniques including Method of Moment technique which was developed by Harrington [10]. The method provides us with approximate solution to the unknown current on antenna of any geometrical shape. It converts into matrix form problem of the form

\[ L(g) = h \]  

provided L is an operator, g is the unknown source quantity, and h is the known excitation. To obtain the unknown quantity in eqn. (1), g is cast in terms of known basis function \( b_n \) and unknown coefficient \( g_n \) of the form

\[ g = \sum_{n=1}^{N} b_n g_n \]  

Substitution of eqn. (2) in eqn. (1) leads to an expression of the form

\[ \sum_{n=1}^{N} b_n L(g_n) = h \]  

This provides a matrix form problem which is subjected to use of Numerical methods.
The expansion of eqn. (3) leads to one dimensional equation which perhaps is not enough to determine the unknown source quantity. A set of linearly independent equations is required and this is achieved by taking the inner product of eqn. (3) with the weighting function \( J_m \) which results to an expression of the form

\[
\langle J_m, h \rangle = b_m \sum_{n=1}^{N} \langle J_m, L_\text{ns}(g_n) \rangle \quad m = 1, 2, 3, \ldots, N \tag{4}
\]

Equation (4) can be recast into matrix equation of the form

\[
[z_{mn}] \{b\} = [v_n] \tag{5}
\]

provided \([z_{mn}] = \langle J_m, L_\text{ns}(g_n) \rangle\) is a matrix formed by the inner product of weighting function and operator on the basis function, \([b]\) is the unknown coefficient and \([v_n] = \langle J_m, h \rangle\) is the known excitation.

Expansion of eqn. (5) leads to expression of the form

\[
\begin{bmatrix}
z_{11} & z_{12} & \cdots & z_{1n} \\
\vdots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & z_{mn}
\end{bmatrix} \begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_n
\end{bmatrix} = \begin{bmatrix}
v_1 \\
v_2 \\
\vdots \\
v_n
\end{bmatrix}
\tag{6}
\]

which indicates that \([z_{mn}]\) is a \(m \times n\) square matrix while \([b_n]\) and \([v_n]\) are column matrices of \(n \times 1\) and \(m \times 1\) dimensions, respectively. Evidently, an inversion of \([z_{mn}]\) and multiplication by \([v_n]\) leads to the complete solution of the quantity of interest. That is,

\[
[b_n] = [z_{mn}]^{-1}[v_n] \tag{7}
\]

The paper proceeds to utilize the method in determining the current distribution on Yagi-Uda antenna. This is discussed in ensuing section.

3. METHODOLOGY

Figure (1) shows the basic geometry of Yagi-Uda antenna of perfectly conducting thin wires, excited by delta gap voltage source and whose radius \(a\) is much smaller compared with the length \(l\) and wavelength \(\lambda\). The figure illustrates an array of the reflector, the driven element and a number of directors, arranged parallel to the \(z\)-axis.

![Geometry of Yagi-Uda array of thin wires](image)

The tangential component of impressed field \(E'\) that impinges on the driven element of the antenna admits an expression in terms of axial components of magnetic vector potential \(A\) and electric scalar potential \(\phi\) in the form rendered as

\[
E'_i = j \omega A_i + \frac{\partial \phi_i}{\partial t} \tag{8}
\]

provided that

\[
A_i = \frac{j \mu_0}{4\pi} \int_0^l (l' \exp[-jk_i R]/R \, dl' \tag{9}
\]

and

\[
\phi_i = -\frac{1}{j \epsilon_0 \omega \mu_0} \frac{\partial}{\partial t} \int_0^l (l' \exp[-jk_i R]/R \, dl' \tag{10}
\]

in which \((\omega, \epsilon_0, \mu_0)\) represent angular frequency, permittivity and permeability of free space, respectively, \(k_0\) is the free space propagation constant, \(I(l')\) is the filamentary current along the axis of the wire, \(\exp[-jk_i R]/R\) is the free space green’s function, \(h\) is \(1/2\) of length of dipole while \(l'\) is the filamentary running variable.

Substitution of eqns. (9) and (10) in eqn. (8) leads to an expression for the tangential component of incident field \(E'_i\) represented as integral equation in term of the unknown current \(I(l')\) in the form rendered as

\[
E'_i = \frac{j \mu_0}{k_0} \int_0^l I(l') \left(1 + jk_i R(2R^2 - 3a^2) + k_i^2 R^2 a^2 \right) e^{-jk_i R}(4\pi R^2) \, dl' \tag{11}
\]

in which

\[
R = \sqrt{R^2 + (l-l')^2} \tag{12}
\]

Equation (11) is known as electric field integral equation (EFIE) and can be written in the form represented by

\[
E'_i = L\left[I(l')\right] \tag{13}
\]

where,

\[
L\left[I(l')\right] = \frac{j \mu_0}{k_0} \int_0^l I(l') \left[1 + jk_i R(2R^2 - 3a^2) + k_i^2 R^2 a^2 \right] e^{-jk_i R}(4\pi R^2) \, dl' \tag{14}
\]

To determine the current distribution on Yagi-Uda antenna, the unknown current \(I(l')\) in eqn. (14) is expanded in terms of unknown complex coefficient \(I_m\) and known expansion function \(J_m\) of the form represented by

\[
I(l') = \sum_{m=1}^{N} \sum_{s=1}^{S} I_m J_m \tag{15}
\]

provided \(n = 1, 2, 3, \ldots, N, \quad s = 1, 2, 3, 4, \ldots, S\). \(N, S\) represent number of elements in the array and number of expansion functions, respectively. It is of interest to note that the basis function in eqn. (15) is an entire domain function of the form

\[
\cos \left(\frac{(2s-1)\pi l'}{2h_n}\right)
\]

Substituting eqn. (15) in eqn. (13) and invoking the linearity property of operator \(L\) yields an expression of the form
\[ E_i = \sum \sum I_{a} L(J_{a}) \quad (16) \]

Equation (16) is not sufficient to determine the current distribution of interest, hence an inner product of eqn. (16) is taken with dirac delta weighting function \( f_a = \delta(I_a - I_a) \).

That is
\[
\langle f_a E_i \rangle = \sum \sum I_{a} \langle f_a L(J_{a}) \rangle \quad (17)
\]
in which \( m \) is the discrete points at which boundary condition is enforced.

Equation (14) is recast into compact equation of the form
\[
[V_a] = [I_a][Z_{n,k,l,m,n}] \quad (18)
\]
in which \([V_a]\) represents voltage matrix due to excitation of the array by an impressed field modeled by delta gap source. Consequently, entries into the voltage matrix are zeros except at the feed point. \([Z_{n,k,l,m,n}]\) is the impedance matrix of the array while \([I_a]\) is the current coefficient of interest.

Consequent upon substitution into eqn. (15) an inverse of the impedance matrix and subsequent multiplication by voltage matrix yields current distribution on Yagi-Uda array. That is
\[
I(l) = \sum \sum J_{a}[Z_{n,k,l,m,n}]^{-1}[V_a] \quad (19)
\]

Equation (19) is then employed to generate numerical results for the current on elements of two forms of Yagi-Uda arrays which are discussed in the subsequent sections.

4. NUMERICAL RESULTS

For the computation of numerical results that depict the features of current on elements of Yagi-Uda antenna, two versions of Yagi-Uda antenna are used; one consists of twelve elements operating at 665 MHz and the other consists of fourteen elements, operating at 200 MHz respectively.

4.1 Design Parameter of twelve element Yagi-Uda array

Utilizing eqn. (19) and design parameters given below, the characteristics profiles of current distribution on elements of 12-element Yagi-Uda array are obtained and portrayed in Fig. (2) which depicts graphical representation of the plot of magnitude of the current as a function of element length.

Length of reflector = 0.51\( \lambda \), length of driven element = 0.5\( \lambda \), length of 10 directors = 0.43\( \lambda \), spacing between reflector and driven element = 0.28\( \lambda \), spacing between driven element and first director = 0.31\( \lambda \), spacing between directors = 0.31\( \lambda \), radius of wire = 0.0325\( \lambda \) each, frequency of propagation = 665 MHz

We observe in those illustrations that current distributions on elements of Yagi-Uda array are symmetrical about the length of each element. It is also found that the currents at the ends of the elements are zero, which is what is expected. Furthermore, it is evident in Fig. (2) that the magnitude of the current is maximum on the driven element.
Moreover, a sinusoidal form of current distribution is observed on elements of the array which agrees well with the assumption used by [4] and [5] while determining the response characteristics of gain of Yagi-Uda array.

We now see how current behaves when the number of elements increases from twelve to fourteen and the frequency of operation changes as well. This is what is discussed in section 4.2.

### 4.2 Design Parameter of fourteen element Yagi-Uda array

Computation of eqn. (19) and using the dimension of 14-element Yagi array stated below leads to numerical results for the current profiles on elements of 14-element Yagi-Uda array which are depicted in Fig. (3). In contrast to the last section, the frequency of operation used is 200MHz.

- Length of reflector \(=0.51\lambda\)
- Length of driven element \(=0.5\lambda\)
- Length of 12 directors \(=0.43\lambda\)
- Spacing between reflector and driven element \(=0.28\lambda\)
- Spacing between driven element and first director \(=0.31\lambda\)
- Spacing between directors \(=0.31\lambda\)
- Radius of wire \(=0.0325\lambda\) each
- Frequency of propagation = 200MHz
We see from Fig. (3), that characteristics profiles for the currents on elements of 14-element Yagi-Uda antenna are to a larger extent similar to those of 12-element Yagi-Uda antenna depicted in Fig. (2), which have been explicitly discussed in section 4.1. This suggests that the nature of current distribution on Yagi-Uda antenna is invariant to changes in the number of elements and frequency of propagation.

5. CONCLUSION

In this paper, the current distributions on elements of two designs of Yagi-Uda antenna are determined using method of moments technique. One model comprises a reflector, driven element and ten directors while the other consists of a reflector, driven element and twelve directors, both operate at 665MHz and 200MHz, respectively. It is found that, the current distribution on elements of those arrays is symmetrical about the length in each instance. Also, it is observed that the current profiles exhibit sinusoidal type of waveform which indicates that, the assumption of sinusoidal current distribution used in the literature is not far from the truth. Furthermore, the magnitude of the current is maximum on the driven element with rather smaller values of current magnitudes existing on directors and reflectors (parasitic elements). In addition, it is discovered that, the currents at the end of the element are zeros and they depict features that do not change in form when there are changes in frequency of propagation and number of elements.

6. REFERENCES