Prime Cordial Labeling

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ABSTRACT
We show that some special families of graphs have prime cordial labeling. We prove that if G is not a prime cordial graph of order m then G ∪ Kₙₐ is a prime cordial graph if E(G) = n - 1, n or n + 1, and we prove that S'(Kₙₐ). Jelly fish graph , Jewel graph, the graph obtained by duplicating a vertex vᵦ in the rim of the helm Hₙ, and the graph obtained by fusing the vertex u₁ with u₃ in a Helm graph Hₙ are prime cordial graphs.

Keywords
Cordial labeling, prime cordial labeling.

1. INTRODUCTION
In this paper we deal with only finite simple and undirected graphs. We shall use the basic notations and terminology of graph theory and number theory as given in [6],[3]. The notion of a prime cordial labeling was introduced by Sundaram, Ponraj and Somasundaram [8]. A prime cordial labeling of a graph G with vertex set V is a bijection f from V to {1, 2, ..., |V|} such that if each edge uv is assigned the label 1 if gcd(f(u); f(v)) = 1 and 0 if gcd(f(u); f(v)) > 1, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. Sundaram, Ponraj, and Somasundaram [8] prove the following graphs are prime cordial: Cₙ if and only if n ≥ 6; Pₙ if and only if n ≠ 3 or 5; Kₙₐ(n odd); the graph obtained by subdividing each edge of Kₙₐ if and only if n ≥ 3; bistars; dragons; jellyfishes; central lemons with three triangles; ladders; Kₙ; if n is even and there exists a prime p such that 2p < n + 1 < 3p; Kₙ; if n is even and if there exists a prime p such that 3p < n + 2 < 4p, and Kₙ; if n is odd and if there exists a prime p such that 5p < n + 3 < 6p. They also prove that if G is a prime cordial graph of even size, then the graph obtained by identifying the central vertex of Kₙₐ with the vertex of G labeled with 2 is prime cordial, and if G is a prime cordial graph of odd size, then the graph obtained by identifying the central vertex of Kₙₐ with the vertex of G labeled with 2 is prime cordial. [10] Vaidya and Shah prove that the following graphs are prime cordial: split graphs of Kₙ; and Bₙ;: the square graph of Bₙ;: the middle graph of Pₙ for n ≥ 4; and Wₙ if and only if n ≥ 8. Also Vaidya and Shah [11],[12] proved following graphs are prime cordial: gear graphs Gₙ for n ≥ 4; helms; closed helms CHₙ for n ≥ 5; ower graphs Fₙ for n ≥ 4; degree splitting graphs of Pₙ and the bistar Bₙ;: double fans Dₙ for n = 8 and n ≥ 10; the graphs obtained by duplication of an arbitrary rim edge by an edge in Wₙ where n ≥ 6; and the graphs obtained by duplication of an arbitrary spoke edge by an edge in wheel Wₙ where n = 7 and n ≥ 9. S. Babitha et al. [1] proved Gₙ Kₙₐ, Gₙ Pₙ, Gₙ Fₙ, Kₙₐ, Kₙₐ Pₙ (n, m > 2), Pₙ (n ≥ 4) are prime cordial graphs.

2. RESULTS
2.1 Definition: A prime cordial labeling of a graph G with vertex set V is a bijection f from V to {1, 2, ..., |V|} such that if each edge uv is assigned the label 1 if gcd(f(u); f(v)) = 1 and 0 if gcd(f(u); f(v)) > 1, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.

2.2 Theorem: Let G be a prime cordial graph with order n and Kₙₐ be the well-known bipartite graph. We have two cases:
(i) If n is an odd number and m is an even number then the disjoint union of G and Kₙₐ is a prime cordial graph.
(ii) If n is an even number , e₁(0) ≥ e₁(1), where e₁(0), e₁(1) are the number of edges labeled with 0 and the number of edges labeled with 1 respectively, and m is an odd number then the disjoint union of G and Kₙₐ is a prime cordial graph.

Proof: Let f: V(G) → {1, 2, ..., n} be given as in Definition 2.1 and u₁, u₂, ..., uₘ be the vertices of G and v₁, v₂, ..., vₘ be the vertices of Kₙₐ. Now we define the new graph called G₁ as the disjoint union of G and Kₙₐ with vertex set V(G₁) = V(G) ∪ {vᵢ : 0 ≤ i ≤ m} and edge set E₁(G₁) = E(G) ∪
Consider the bijective function \( g \) defined by

\[ g(v_i) = \alpha, \text{ where } \alpha \text{ is the greatest even number such that } n + 1 \leq \alpha \leq n + m + 1. \]

\[ g(v_i) = n + i; 1 \leq i \leq m, i \neq \alpha \]

Now we have

\[ g(v_0v_{2i-1}) = 1; \text{for } 1 \leq i \leq \frac{m}{2} \text{ and } g(v_0v_{2i}) = 0; 1 \leq i \leq \frac{m}{2}. \]

Hence the total number of edges labeled 1's are given by

\[ e(1) = \frac{m}{2} \]

and the total number of edges labeled 0's are given by

\[ e(0) = \frac{m}{2}. \]

Since \( G \) is prime cordial of odd order then
e_1 = \left\lceil \frac{n}{2} \right\rceil \text{ and } e_0 = \left\lfloor \frac{n}{2} \right\rfloor \text{ or } e_1 = \left\lfloor \frac{n}{2} \right\rfloor \text{ and } e_0 = \left\lceil \frac{n}{2} \right\rceil \].

Hence the total number of edges labeled with 1's of the graph \( G_1 \) is

\[ e_g(1) = \frac{n}{2} + \frac{m}{2} \]

and the total number of edges labeled with 0's is

\[ e_g(0) = \frac{n}{2} + \frac{m}{2} \]

or

\[ e_g(1) = \frac{n}{2} + \frac{m}{2} \]

and

\[ e_g(0) = \frac{n}{2} + \frac{m}{2}. \]

Then \( G_1 \) is a prime cordial graph. (See Figure 1.)

**Case (ii)** \( g(u_i) = f(u_i); 1 \leq i \leq n-1 \)

\[ g(v_i) = n + i; 1 \leq i \leq m \]

\[ g(u_i) \text{ we interchange the vertex labeled } 2 \text{ in } G \text{ by } \alpha, \text{ where } \alpha \text{ is the greatest even number such that } n + 1 \leq \alpha \leq n + m + 1 \]

Now we have

\[ g(v_0v_{2i-1}) = 1; \text{for } 1 \leq i \leq \frac{m}{2} \text{ and } g(v_0v_{2i}) = 0; 1 \leq i \leq \frac{m}{2}. \]

Hence the total number of edges labeled 1's of \( K_{1,m} \) are given by

\[ \left\lceil \frac{m}{2} \right\rceil \]

and the total number of edges labeled 0's of \( K_{1,m} \) are given by

\[ \left\lfloor \frac{m}{2} \right\rfloor - 1. \]

Now since \( G \) is prime cordial of even order and

\[ e_1(0) \geq e_1(1), \text{either we have } e_0(0) > e_1(1) \text{ then } e_1(1) = \frac{n}{2} - 1, \]

\[ e_0(0) = \frac{n}{2} \text{ or } e_1(0) = e_1(1) \text{ then } e_1(1) = \frac{n}{2}, e_0(0) = \frac{n}{2} \]

hence the total number of edges labeled with 1's of \( G_1 \) if

\[ e_0(1) \geq e_1(1) \]

is

\[ e_0(1) = \left\lceil \frac{n}{2} - \frac{m}{2} \right\rceil \text{ and } e_0(0) = \left\lfloor \frac{n}{2} + \frac{m}{2} \right\rfloor. \]

\[ \text{then } |e_0(1) - e_0(0)| = \left\lfloor \frac{n}{2} - 1 + \frac{m}{2} \right\rfloor - 1, \]

\[ 0 < 1. \text{or if } e_0(0) \geq e_1(0) = e_1(1) \]

\[ e_0(0) = \left\lceil \frac{n}{2} + \frac{m}{2} \right\rceil - 1, \text{then } |e_0(1) - e_0(0)| = \left\lfloor \frac{n}{2} + \frac{m}{2} \right\rfloor - \left\lceil \frac{n}{2} + \frac{m}{2} \right\rceil = 1. \]

Then \( G_1 \) is a prime cordial graph.

**2.3 Example**

**2.4 Theorem:** If \( G \) is not a prime cordial graph of order \( m \) then \( G \cup K_{1,n} \) is a prime cordial graph if

\[ E(G) = n - 1, n \text{ or } n + 1 \]

**Proof:** Let \( a_1 < a_2 < \cdots < a_m \) be the first \( m \) even numbers less than or equal to \( m+n+1 \). We label the vertices of \( K_{1,n} \) by \( \{1, 2, \ldots, m+n+1\} \setminus \{a_1, a_2, \ldots, a_m\} \) such that center of \( K_{1,n} \) is labeled by 1 and we label the vertices of \( G \) by \( a_1, a_2, \ldots, a_m \) then we find that \( G \cup K_{1,n} \) is prime cordial graph.
2.5 Example

3. SOME FAMILIES OF PRIME CORDIAL GRAPHS

3.1 Definition [4]: Jelly fish graphs $f(m, n)$ are obtained from a 4-cycle $v_1, v_2, v_3, v_4$ by joining $v_1$ and $v_3$ with an edge and appending $m$ pendent edges to $v_2$ and $n$ pendent edges to $v_4$.

3.2 Theorem: $f(n,n)$ is a prime cordial graph.

Proof: Let $V(f(n,n)) = \{v_1, v_2, v_3, v_4, u_0, w_i; 1 \leq i \leq n\}$ and $E(G) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1, v_1v_5, v_2u_0, v_4w_i; 1 \leq i \leq n\}$

Then $|V(G)| = 2n + 4$ and $|E(G)| = 2n + 5$.

Define $f : V(G) \rightarrow \{1, 2, 3, ..., 2n + 4\}$ as follows:

\[
f(v_i) = 2n + 4, f(v_2) = 2, f(v_4) = 1
\]

$f(v_2)$ is the smallest prime number such that it divides $2n + 4$

$f(u_i) = 4 + 2(i - 1), 1 \leq i \leq n$

For the vertices $w_1, ..., w_n$, we assign the remaining odd numbers which are less than $2n + 4$.

Then $e_f(0) = n + 2$ and $e_f(1) = n + 3$.

Therefore, $|e_f(0) - e_f(1)| \leq 1$. Hence $f(n,n)$ is prime cordial graph.

3.3 Example

\[
K_3 \cup K_{1,4}
\]

Fig 3

\[
K_3 \cup K_{1,2}
\]

Fig 3

\[
K_5 \cup K_{1,10}
\]

Fig 3

3.4 Definition [4]: For a graph $G$ the splitting graph $S'(G)$ of a graph $G$ is obtained by adding for each vertex $v$ of $G$ a new vertex $v'$ so that $v'$ is adjacent to every vertex that is adjacent to $v$.

3.5 Theorem: The graph $S'(K_{2,n})$ is prime for all $n \neq 2$.

Proof: Let $x_1, x_2, v_1, v_2, ..., v_n$ be the vertices of $K_{2,n}$.

Then $x_1, x_2, v_1, v_2, ..., v_n, x'_1, x'_2, v'_1, v'_2, ..., v'_n$ are the vertices of $S'(K_{2,n})$ and then $|V(G)| = 2n + 4$ and $|E(G)| = 6n$.

Define $f : V(G) \rightarrow \{1, 2, 3, ..., 2n + 4\}$ as follows:

For $n = 5$

\[
f(x_1) = 2, f(x_2) = 14
\]

\[
f(v_i) = 4 + 2(i - 1), 1 \leq i \leq 5
\]

\[
f(x'_1) = 3, f(x'_2) = 9
\]

\[
f(v'_1) = 1, f(v'_2) = 5, f(v'_3) = 7, f(v'_4) = 11, f(v'_5) = 13
\]

Fig 5: prime cordial labeling for $S'(K_{2,5})$
For all $n \neq 2,5$

$f(x_1) = 3, f(x_2) = 5$

$f(v_i) = 6 + 2(i - 1), 1 \leq i \leq n$

$f(x_1) = 2, f(x_2') = 4, f(v'_1) = 1, f(v'_2) = 7$

$f(v'_i) = 5 + 2(i - 1), 3 \leq i \leq n$

Then $e_f(0) = e_f(1) = 3n$ for any $n, n \neq 2$.

Therefore, $|e_f(0) - e_f(1)| \leq 1$. Hence $G$ is prime cordial graph.

### 3.6 Example

3.7 Definition [4]: The jewel graph $J_n$ is the graph with vertex set $\{u, v, x_1, x_2, v_i; 1 \leq i \leq n\}$ and edge set $\{ux_1, vx_1, ux_2, vx_2, x_1x_2, uv_i, vv_i; 1 \leq i \leq n\}$.

3.8 Theorem: The Jewel graph $J_n$ is prime cordial graph if $n$ is even.

**Proof:** Let $V(J_n) = \{u, v, x_1, x_2, v_i; 1 \leq i \leq n\}$ and $E(J_n) = \{ux_1, vx_1, ux_2, vx_2, x_1x_2, uv_i, vv_i; 1 \leq i \leq n\}$.

Then $|V(G)| = n + 4$ and $|E(G)| = 2n + 5$.

If $n=2$: $f(u) = 2, f(v) = 3, f(x_1) = 4, f(x_2) = 6, f(v_1) = 1, f(v_2) = 5$

If $n \geq 4$ we define $f : V(G) \to \{1, 2, 3, ..., n + 4\}$ as follows:

$f(u) = 2, f(x_1) = 4$

$f(v)$ is the greatest even number less than or equal to $n+4$ written in the form $2p_1$, where $p_1$ is a prime number $\geq 3$.

*For the vertices $x_2, v_1, v_2, ..., v_{n-1}$* We assign the remaining even numbers which $\leq n + 4$.

For the vertices $v_{n-1}^1, v_2, v_{n-1}^2, ..., v_n$ we assign the labeling $1, 3, 5, ..., n+3$. Now we have two cases

If $f(v_n) = n + 3 \neq 3p_1$, then $gcd(v, v_n) = 1$, so $G$ is prime cordial graph. See Figure 6 (1), (3), (4).

If $f(v_n) = n + 3 = 3p_1$, then $gcd(v, v_n) \neq 1$ so we assign the vertex $v$ as $f(v) = 2p_2$ which $p_1, p_2$ are prime numbers and such that $p_2$ the smallest number immediately following the first $p_1$, see Figures (2).

Then $e_f(1) = n + 2$ and $e_f(0) = n + 3$.

Therefore, $|e_f(0) - e_f(1)| \leq 1$. Hence $J_n$ is prime cordial graph for $n$ is even.

### 3.9 Example

3.8 Theorem: The Jewel graph $J_n$ is prime cordial graph if $n$ is even.

**Proof:** Let $V(J_n) = \{u, v, x_1, x_2, v_i; 1 \leq i \leq n\}$ and $E(J_n) = \{ux_1, vx_1, ux_2, vx_2, x_1x_2, uv_i, vv_i; 1 \leq i \leq n\}$.

Then $|V(G)| = n + 4$ and $|E(G)| = 2n + 5$.

If $n=2$: $f(u) = 2, f(v) = 3, f(x_1) = 4, f(x_2) = 6, f(v_1) = 1, f(v_2) = 5$

If $n \geq 4$ we define $f : V(G) \to \{1, 2, 3, ..., n + 4\}$ as follows:

$f(u) = 2, f(x_1) = 4$

$f(v)$ is the greatest even number less than or equal to $n+4$ written in the form $2p_1$, where $p_1$ is a prime number $\geq 3$.

*For the vertices $x_2, v_1, v_2, ..., v_{n-1}$* We assign the remaining even numbers which $\leq n + 4$.

For the vertices $v_{n-1}^1, v_2, v_{n-1}^2, ..., v_n$ we assign the labeling $1, 3, 5, ..., n+3$. Now we have two cases

If $f(v_n) = n + 3 \neq 3p_1$, then $gcd(v, v_n) = 1$, so $G$ is prime cordial graph. See Figure 6 (1), (3), (4).

If $f(v_n) = n + 3 = 3p_1$, then $gcd(v, v_n) \neq 1$ so we assign the vertex $v$ as $f(v) = 2p_2$ which $p_1, p_2$ are prime numbers and such that $p_2$ the smallest number immediately following the first $p_1$, see Figures (2).

Then $e_f(1) = n + 2$ and $e_f(0) = n + 3$.

Therefore, $|e_f(0) - e_f(1)| \leq 1$. Hence $J_n$ is prime cordial graph for $n$ is even.
3.10 Definition [5]: Duplication of a vertex \( v_k \) of a graph \( G \) produces a new graph \( G_k \) by adding a vertex \( v_k' \) with \( N(v_k') = N(v_k) \) (the set of neighbor vertices to \( v_k' \)) . In other words a vertex \( v_k' \) is said to be a duplication of \( v_k \) if all the vertices which are adjacent to \( v_k \) are now adjacent to \( v_k' \).

3.11 Theorem: The graph obtained by duplicating a vertex \( v_k \) in the rim of the helm \( H_n \) is a prime cordial graph.

Proof: Let \( V(H_n) = \{ u, u_1, u_2, ..., u_n, v_1, v_2, ..., v_n \} \).
\( E(H_n) = \{ uu_i; 1 \leq i \leq n \} \cup \{ u_i v_i; 1 \leq i \leq n \} \cup \{ u_{i+1} v_i; 1 \leq i \leq n - 1 \} \cup u_1 u_n \)

Let \( G_k \) be the graph obtained by duplicating the vertex \( u_k \) in \( H_n \) and let the new vertex be \( u_k' \).

Then \( |V(G_k)| = 2n + 2 \) and \( |E(G_k)| = 3n + 4 \).

Define a labeling \( f: V(G_k) \rightarrow \{ 1, 2, ..., 2n + 2 \} \) as follows:

\[
\begin{align*}
\text{Case(1): } & \text{If } n \text{ is even, } n \geq 10, n \neq 14 + 6k, k \geq 0 \\
& f(u) = 2, \quad f(u_1) = 10, \quad f(u_2) = 8, \quad f(u_3) = 4 \\
& f(u_{i+1}) = 12 + 2(i - 1), 1 \leq i \leq \frac{n}{2} - 4 \\
& f(u_{2i+1}) = 6 \\
& f(u_{2i+2}) = 3 \\
& f(u_{n-2i}) = 5 + 4i, 0 \leq i \leq \frac{n}{2} - 2 \\
& f(v_i) = 2n + 2 + 2(1 - i), 1 \leq i \leq \frac{n}{2} \\
& f(v_{n-2i}) = 7 + 4i, 0 \leq i \leq \frac{n}{2} - 2 \\
& f(u^{n}_{n+1}) = 2n + 1 \\
& f(u^2_{n+1}) = 1
\end{align*}
\]
Case (2): $n = 14 + 6k$, $k \geq 0$

$$f(u) = 2, f(u_4) = 10, f(u_2) = 8, f(u_9) = 4$$

$$f(u_{i+3}) = 12 + 2(i - 1), 1 \leq i \leq \frac{n}{2} - 4$$

$$f\left(\frac{u_m}{2}\right) = 6, f\left(\frac{u_{m+1}}{2}\right) = 3$$

$$f(u_{n-i}) = 5 + 4i, 0 \leq i \leq \frac{n}{2} - 2$$

$$f(v_i) = 2n + 2 + 2(1 - i), 1 \leq i \leq \frac{n}{2}$$

$$f(v_{n-i}) = 7 + 4i, 0 \leq i \leq \frac{n}{2} - 3$$

$$f\left(\frac{v_{2+i}}{2}\right) = 2n - 1, f\left(\frac{v_{2+i}}{2}\right) = 2n + 1$$

$$f(u_{12}) = 1$$

Case (3): $n$ is odd, $n \geq 9, n \neq 13 + 6k, k \geq 0$

$$f(u) = 2, f(u_4) = 10, f(u_2) = 8, f(u_9) = 4$$

$$f(u_{i+3}) = 12 + 2(i - 1), 1 \leq i \leq \frac{n-1}{2} - 3$$

$$f\left(\frac{u_{n+i}}{2}\right) = 6, f\left(\frac{u_{n+i+1}}{2}\right) = 3$$

$$f(u_{n-i}) = 5 + 4i, 0 \leq i \leq \frac{n-1}{2} - 2$$

$$f(v_i) = 2n + 2 + 2(1 - i), 1 \leq i \leq \frac{n-1}{2}$$

$$f(v_{n-i}) = 7 + 4(i - 1), 1 \leq i \leq \frac{n-1}{2}$$

$$f\left(\frac{v_{2+i}}{2}\right) = 2n - 1, f(u_9) = 1$$

Case (4): $n = 13 + 6k$, $k \geq 0$

$$f(u) = 2, f(u_4) = 10, f(u_2) = 8, f(u_9) = 4$$

$$f(u_{i+3}) = 12 + 2(i - 1), 1 \leq i \leq \frac{n-1}{2} - 3$$

$$f\left(\frac{u_{n+i}}{2}\right) = 6$$

$$f(u_{n-i}) = 5 + 4i, 0 \leq i \leq \frac{n-1}{2} - 1$$

$$f(v_i) = 3$$

$$f(v_{n-i}) = 7 + 4(i - 1), 1 \leq i \leq \frac{n-1}{2}$$

$$f\left(\frac{v_{2+i}}{2}\right) = 2n - 1, f(u_9) = 1$$
$f(u_{11}^+)=1$

**Fig13: prime cordial labeling for duplicating**

the vertex $u_{11}$ in $H_{13}$

Then when $n$ is even and $n \geq 10, n \neq 14 + 6k, k \geq 0$

then $e_f(0) = e_f(1) = \frac{3n+4}{2}$ also if $n = 14 + 6k$ then $e_f(0) = e_f(1) = \frac{3n+4}{2}$, when $n$ is odd and

$n \neq 13 + 6k, k \geq 0$ then $e_f(1) = \left\lceil \frac{3n+4}{2} \right\rceil$ and $e_f(0) = \left\lfloor \frac{3n+4}{2} \right\rfloor$ and if $n = 13 + 6k$ then $e_f(1) = \left\lceil \frac{3n+4}{2} \right\rceil$ and $e_f(0) = \left\lfloor \frac{3n+4}{2} \right\rfloor$

Therefore, $|e_f(0) - e_f(1)| \leq 1$. Hence $G$ is prime cordial graph.

3.12 **Definition:** Let $u$ and $v$ be two distinct vertices of a graph $G$. A new graph $G'$ is constructed by identifying (fusing) two vertices $u$ and $v$ by a single vertex $x$ such that every edge which was incident with either $u$ or $v$ in $G$ is now incident with $x$ in $G$.

3.13 **Theorem:** The graph obtained by fusing the vertex $u_4$ with $u_3$ in a Helm graph $H_n$ is prime cordial graph.

**Proof:** Let $V(H_n) = \{u, u_1, u_2, ..., u_{2n}, v_1, v_2, ..., v_n\}$, $E(H_n) = \{u_iu; 1 \leq i \leq n\} \cup \{u_iv; 1 \leq i \leq n\} \cup \
\{u_iu_{i+1}; 1 \leq i \leq n - 1\} \cup u_1u_n$

Let $G_3$ be the graph obtained by fusing the vertex $u_4$ with $u_3$ in $H_n$.

Then $|V(G_3)| = 2n$ and $|E(G_3)| = 3n - 2$. 

Define a labeling $f: V(G_3) \rightarrow \{1, 2, ..., 2n\}$ as follows:
Case(1): $n$ is even, $n \geq 12$, $n = 12 + 6k$, $k \geq 0$

- $f(u) = 2$, $f(u_1) = 10$, $f(u_2) = 4$, $f(u_4) = 8$
- $f(u_{i+k}) = 12 + 2(i - 1)$, $1 \leq i \leq \frac{n-2}{2} - 4$
- $f\left(\frac{u_n}{2}\right) = 6$, $f\left(\frac{u_n}{2} + 1\right) = 3$
- $f(u_{n-i}) = 5 + 4i$, $0 \leq i \leq \frac{n}{2} - 2$
- $f(v_1) = 2n$, $f(v_2) = 2n - 2$
- $f(v_{i+k}) = 2n - 4 - 2i$, $0 \leq i \leq \frac{n}{2} - 3$
- $f(v_{n-i}) = 7 + 4i$, $0 \leq i \leq \frac{n}{2} - 2$
- $f(v_1') = 1$

**Fig. 14: prime cordial labeling for fusion**

of $u_1$ and $u_3$ in $H_4, H_5, ..., H_{10}$

Case(2): $n = 14, 16, 20, 22, ...$

- $f(u) = 2$, $f(u_1) = 10$, $f(u_2) = 4$, $f(u_4) = 8$
- $f(u_{i+k}) = 12 + 2(i - 1)$, $1 \leq i \leq \frac{n-1}{2} - 5$
- $f\left(\frac{u_n}{2}\right) = 6$, $f\left(\frac{u_n}{2} + 1\right) = 3$
- $f(u_{n-i}) = 5 + 4i$, $0 \leq i \leq \frac{n}{2} - 2$
- $f(v_1) = 2n - 2$, $f(v_2) = 2n - 4$
- $f(v_{i+k}) = 2n - 6 - 2i$, $0 \leq i \leq \frac{n}{2} - 4$
- $f(v_{n-i}) = 7 + 4i$, $0 \leq i \leq \frac{n}{2} - 2$
- $f(v_1') = 2n$
- $f\left(\frac{v_{n+1}}{2}\right) = 1$

**Fig. 16: prime cordial labeling for fusion**

of $u_1$ and $u_3$ in $H_{14}$

Case(3): $n$ is odd

- $f(u) = 2$, $f(u_1) = 10$, $f(u_2) = 4$, $f(u_4) = 8$
- $f(u_{i+k}) = 12 + 2(i - 1)$, $1 \leq i \leq \frac{n-1}{2} - 4$
- $f\left(\frac{u_n}{2}\right) = 6$
- $f(u_{n-i}) = 5 + 4i$, $0 \leq i \leq \frac{n-1}{2} - 1$
- $f(v_1) = 2n - 2$, $f(v_2) = 2n - 4$
- $f(v_{i+k}) = 2n - 6 - 2i$, $0 \leq i \leq \frac{n-1}{2} - 4$
- $f\left(\frac{v_{n+1}}{2}\right) = 3$
- $f(v_{n-i}) = 7 + 4i$, $0 \leq i \leq \frac{n-1}{2} - 2$
- $f\left(\frac{v_{n+1}}{2}\right) = 2n$
- $f(v_1') = 1$
Then when n is even $e_f(0) = e_f(1) = \frac{3n-2}{2}$, when n is odd $e_f(0) = \left\lfloor \frac{3n-2}{2} \right\rfloor$, $e_f(1) = \left\lfloor \frac{3n-2}{2} \right\rfloor + 1$

Therefore, $|e_f(0) - e_f(1)| \leq 1$. Hence $G$ is prime cordial graph.

**3.14 Theorem:** The graph $G$ which is obtained by attaching central vertex of a star $K_{1,n}$ at one of the vertices of $C_3$ is prime cordial graph.

**Proof:** Let $V(G)=\{u, v, w, u_i; 1 \leq i \leq n\}$ and $E(G) = \{uv, vw, wu\} \cup \{ui; 1 \leq i \leq n\}$

Then $|V(G)| = n + 3$ and $|E(G)| = n + 3$.

We define $f : V(G) \rightarrow \{1, 2, 3, ..., n + 3\}$ as:

- $f(u) = 2$, $f(v) = 6$, $f(w) = 3$
- $f(u_1) = 1$, $f(u_2) = 4$, $f(u_3) = 5$
- $f(u_i) = 4 + (i - 1)$, $4 \leq i \leq n$

If n is odd then $e_f(1) = \frac{n+3}{2}$, $e_f(0) = \left\lfloor \frac{n+3}{2} \right\rfloor$; if n is even then

$e_f(0) = \left\lfloor \frac{n+3}{2} \right\rfloor$, $e_f(1) = \left\lfloor \frac{n+3}{2} \right\rfloor + 1$

Therefore, $|e_f(0) - e_f(1)| \leq 1$. Hence $G$ is prime cordial graph.

**Example3.15.**

**Fig17:** prime cordial labeling for fusion of $u_1$ and $u_3$ in $H_{11}$

**Fig18:** prime cordial labeling for attaching central vertex of the star $K_{1,6}$ at one of the vertices of $C_3$

**Fig19:** prime cordial labeling for attaching central vertex of a star $K_{1,7}$ at one of the vertices of $C_3$

**4. CONCLUSION**

If $G$ is a prime cordial graph with order n and $K_{1,m}$ be the well-known bipartite graph. We have two cases:

(i) If n is an odd number and m is an even number then the disjoint union of $G$ and $K_{1,m}$ is a prime cordial graph.

(ii) If n is an even number, $e_f(0) \geq e_f(1)$, where $e_f(0), e_f(1)$ are the number of edges labeled with 0 and the number of edges labeled with 1 respectively, and m is an odd number then the disjoint union of $G$ and $K_{1,m}$ is a prime cordial graph.

In the future we will try to construct other general theorems and find some other prime cordial families.
5. REFERENCES


