2-D Near Field Source Localization by Evolutionary Technique Exploiting the L-Type Geometry of Sensor Array

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ABSTRACT
The efficiency of two dimensional (2-D) Direction of Arrival (DOA) estimation relies on the geometry of array. Among many array geometries, L-type array structure is becoming more popular among researchers because it can be decoupled into two uniform linear arrays (ULAs) and require less number of array elements as compared to other planar arrays. This paper propose a novel, fast and low complexity method for joint estimation of range and 2-D DOAs (elevation and azimuth) of near field sources. The main focus of this paper is to present the efficacy of performance and ease in implementation of L-type array structure when it is integrated with Differential Evolution (DE), a global evolutionary optimizer. To avoid the pair matching of estimated parameters, mean square error is used as fitness evaluation function as it only requires a single snapshot of array output to achieve optimal convergence. The robustness of proposed method is tested by a large number of computer simulations and statistical performance analysis is compared with other techniques.

Keywords
Direction of arrival, 2-D estimation, Evolutionary technique, L-type array, Near field.

1. INTRODUCTION
Over several decades, two-dimensional (2-D) direction of arrival (DOA) estimation has received substantial consideration [1]. Estimation of DOAs in 2-D (i.e. elevation and azimuth angles) of multiple sources using planar arrays with numerous array geometries plays a significant role in various practical applications such as sonar, radar, telecommunication, signal/speech processing etc. [2]. Numerous 2-D array geometries have been developed, such as circular array, planar array, spherical array etc. The use of L-type planar array, composed of two uniform linear arrays (ULAs) attached orthogonally from end of each ULA, is advantageous in terms of coverage area and employment as it needs less number of array elements as compared to rectangular and spherical arrays [3-7]. Another advantage of deploying L-type array is to obtain the 2-D DOAs by decoupling the array into two ULAs and independently estimating DOA for each subarray [8-10]. Hence, L-type array is easy to implement and can provide better estimation performance [4], [11].

As the dimensionality of DOA estimation increases, the computational burden of estimation process is relentlessly affected by the geometry of array [12], [13] and pair-matching of the DOAs become essential, which may result in pair matching inaccuracy and poor angle estimation performance [3], [12], [16-18]. Much efforts have concentrated to reduce the computational burden of 2-D DOA estimation as most pair-matching algorithms comprise of 2-D searching and nonlinear optimization.

The rest of the paper is arranged as follows: section 2 discusses related work, section 3 gives the signal model and array structure; section 4 discusses 2-D near field source localization. Section 5 is dedicated for simulation results, section 6 gives the conclusion and future enhancements, and section 7 and section 8 are dedicated for acknowledgments and references respectively.

2. RELATED WORK
From the last few decades, researchers are using subspace based techniques for source localization (far field or near field) in 1-D and 2-D. The main problem with subspace based methods is that they require enormous number of snapshots of array output for estimation which increases the computational complexity and make the use of pair matching techniques essential. Some of the recent methods for 2-D DOA estimation are discussed which tried to overcome these problems. Some techniques exploit the property of uniform linear subarray and special matrix or tensor techniques such as joint SVD technique [19], parallel factor analysis [20] to achieve automatic pairing. The JSVD method in [19] needs K one dimensional searches for the “beamforming-like” procedure (where K denotes the number of sources) and PARAFAC method in [20] may converge slowly which needs many iterations and results in increase of computational complexity. In [11], first the array aperture is enlarged by the conjugate symmetry property of ULA manifold matrix. Then,
a large array-received-data-like matrix is formed and propagator matrix is obtained by applying propagator method (PM) [21] into the large array-received-data-like matrix. Finally, the angles are estimated with the ESPRIT algorithm [22] utilizing the potential rotational invariance property of array manifold matrix which is constructed with the propagator matrix. The method in [11] can estimate angles without spectrum search or pair matching. Results show that the method in [11] yields better angle estimation performance than classical JSVD [20], PARAFAC [20], CODE [9], and CESA [10] methods and the computation complexity is lower than that of JSVD, PARAFAC, CODE, and CESA methods.

In this fast growing technological era, evolutionary computing such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Genetic Programming (GP) and Differential Evolution (DE) etc. have become very popular and demonstrated their importance and effectiveness [23]. The implementation of these techniques is easy and they perform as potent global optimizers which does not get stuck in local minima. For more accuracy and efficiency, these algorithms can be hybridized with local optimizers such as Active set (AS), Interior point Algorithm (IPA), pattern Search (PS) etc. [24-28].

The purpose of this paper is to jointly estimate the range and 2-D DOAs (i.e. elevation and azimuth angles) of near field narrowband sources by exploiting the geometric structure of L-type array placed in the plane in a computationally effective manner. An evolutionary computing technique named Differential Evolution is applied to optimize the estimation process as it does not need spectrum peak search or additional angle pair matching procedure. The effectiveness of DE with L-type array structure and the theoretical analysis are verified through numerical examples. The simulation results prove that DE with L-type array structure has better estimation with a single snapshot as compared to GA and PSO even at low SNR, which is the verification of the theoretical investigations.

3. SIGNAL MODEL AND ARRAY STRUCTURE

Consider a geometry of two ULAs orthogonal to each other forming an L-type configuration as shown in Fig. 1. One ULA is placed along x-axis consisting of ‘M’ sensors while the second ULA is placed along z-axis consisting of ‘N’ sensors. The element on the origin is owned by z-axis and is taken as reference sensor. The inter-sensor spacing between two consecutive sensors in each sub-array is equaled and set as ‘d’, let $d_x = d_z = \frac{\lambda}{2}$ and $\lambda$ is the wavelength of carrier frequency. Suppose that ‘K’ narrow band near field sources impinge on the L-type sensor array from different directions with elevation and azimuth angles ‘$\theta_k$’ and ‘$\phi_k$’ and ranges ‘$r_k$’ where $k = 1, 2, ... , K$.

Let the number of sources ‘K’ be known and $K \leq M, K \leq N$. If the number of sources is unknown then it can be estimated by methods like Akaike Information Theoretic Criteria (AIC) or Minimum Description Length (MDL). The signal power is taken same for all sensors. This assumption is to lower the simulation complexity and is widely used in literature. Another assumption is that the incident signals and noise are uncorrelated. The elevation and azimuth angles ‘$\theta_k$’ and ‘$\phi_k$’ and ranges ‘$r_k$’ are estimated w.r.t. reference sensor. The signals received on sub-array along x-axis and sub-array along z-axis can be mathematically written as:

$$x_m = \sum_{k=1}^{K} s_k e^{j(\omega_{mk} + m^2 \phi_{mk})} + n_m ; \quad m = 1, ..., M$$  \hspace{1cm} (1)

and

$$z_k = \sum_{k=1}^{K} s_k e^{j(\omega_{nk} + n^2 \phi_{nk})} + n_k ; \quad n = 1, ..., N$$  \hspace{1cm} (2)

where, $\omega_{mk} = - \frac{2\pi d_x}{\lambda} \sin \theta_k$, $\omega_{nk} = - \frac{2\pi d_z}{\lambda} \sin \phi_k$ and $\varphi_{mk} = \pi \frac{d_x^2}{\lambda^2} \cos^2 \theta_k$, $\varphi_{nk} = \pi \frac{d_z^2}{\lambda^2} \cos^2 \phi_k$

In vector form, $x = A_x(\theta, \phi, r) s + n_x$ (3) and $z = A_z(\theta, \phi, r) s + n_z$ (4)

where, $x = [x_1, x_2, ..., x_M]^T$, $z = [x_1, x_2, ..., x_N]^T$, $s = [s_1, s_2, ..., s_K]^T$, $A_x(\theta, \phi, r) = [a_1(\theta, \phi, r) , a_1(\theta, \phi, r), ..., a_1(\theta, \phi, r) ]$, $A_z(\theta, \phi, r) = [a_1(\theta, \phi, r) , a_1(\theta, \phi, r), ..., a_1(\theta, \phi, r) ]$

$$a_1(\theta, \phi, r_k) = [1, e^{j(\omega_{mk} + \varphi_{mk})}, e^{j(2\omega_{mk} + 2^2 \varphi_{mk})}, ..., e^{j((M-1)\omega_{mk} + (M-1)^2 \varphi_{mk})}]^T$$

$$a_2(\theta, \phi, r_k) = [1, e^{j(\omega_{nk} + \varphi_{nk})}, e^{j(2\omega_{nk} + 2^2 \varphi_{nk})}, ..., e^{j((M-1)\omega_{nk} + (M-1)^2 \varphi_{nk})}]^T$$

$$n_x = [n_{x1}, n_{x2}, ..., n_{xN}]^T$$ and $n_z = [n_{z1}, n_{z2}, ..., n_{zK}]^T$ for $k = 1, 2, ..., K$. The steering vectors along x-axis and z-axis are $a_x(\theta_0, \phi_0, r_k)$ and $a_z(\theta_0, \phi_0, r_k)$ respectively and $n_x$ and $n_z$ are the additive white Gaussian noise at the output of sub-arrays along x-axis and z-axis respectively. The parameters i.e. the elevation and azimuth angles of arrival ‘$\theta_k$’ and ‘$\phi_k$’ and the ranges ‘$r_k$’ of sources, for $k = 1, 2, ..., K$ are unknown and to be estimated from the array output.

4. 2-D NEAR FIELD SOURCE LOCALIZATION

4.1 2-D Differential Evolution as global optimizer

**Step 1:** Let $L$ and $H$ be the lower and upper limits of genes (parameters) in a chromosome respectively. Let the total number of chromosomes in one generation is ‘C’, number of genes in any chromosome is ‘G’, and total number of generations is GEN, then

$$\overline{d_{be}} = L + \text{rand()} * (H - L)$$  \hspace{1cm} (5)
where,
\[ c = \text{chromosome number for } 1 \leq i \leq C \]
\[ ge = \text{gene number for } 1 \leq ge \leq G \]
\[ g = \text{generation number } 1 \leq g \leq \text{GEN} \]
\[ rand() = \text{a random number chosen from 0 to 1} \]

The chromosomes in a generation consist of genes in the following order:
\[ \vec{a}^g = [r_1 \cdots r_k \ \theta_1 \cdots \theta_k \ \varphi_1 \cdots \varphi_k]^T \]

The optimizer checks the fitness of all the chromosomes initially created in one generation using:
\[ f(\vec{a}^g) < \varepsilon \quad (6) \]

where,
\[ f(\vec{a}^g) = \| \vec{x} - \vec{x}^{DE} \|^2 + \| \vec{z} - \vec{z}^{DE} \|^2 \quad (7) \]

is the mean square error and ‘ε’ is a very small positive number. \( \vec{x}^{DE} \) and \( \vec{z}^{DE} \) are calculated by using \( \vec{a}^g \) in equations (1) and (2) respectively. The optimizer stops if the above condition is satisfied, otherwise it proceeds to step 2.

**Step 2:** Update all the chromosomes of the present generation ‘g’ from 1 to C and create a new generation of chromosomes ‘g + 1’. The updating process consists of three sub-steps: mutation, crossover and selection. Suppose we pick \( c^{th} \) chromosome \( \vec{a}^{g} \), then

1. **Mutation:**
   Choose three numbers randomly from 1 to C i.e. \( (c_1, c_2, c_3) \), all different and not equal to \( c \). Then,
   \[ \vec{b}^{g+1} = \vec{a}^{c_1} + \vec{F}(\vec{a}^{c_2} - \vec{a}^{c_3}) \]
   creates an intermediate chromosome. Here, \( \vec{F} \) is the scale factor (problem dependent) which can be selected from 0.4 to 1.2 but usually selected as 0.5.

2. **Crossover:**
   \[ \vec{c}^{g+1} = \begin{cases} \vec{c}^{g+1} & \text{if } \text{rand()} \leq \text{CR} \text{ or } ge = \text{ge}_{\text{rand}} \\ \vec{c}^{g} & \text{otherwise} \end{cases} \quad \forall \ge \]
   where, \( \text{rand()} \) is a number randomly selected from 0 to 1, CR (cross-over rate) = 0.5 to 0.9 (generally) and \( \text{ge}_{\text{rand}} \) is chosen randomly from 1 to G.

3. **Selection:**
   The new chromosomes for next generation ‘g + 1’ are selected and replaced with the old chromosomes, if fitness is better, using the following equation:
   \[ \vec{a}^{g+1} = \begin{cases} \vec{c}^{g+1} & \text{if } f(\vec{a}^{g+1}) < f(\vec{a}^{g}) \\ \vec{a}^{g} & \text{otherwise} \end{cases} \]

**Step 3:** If \[ f(\vec{a}^{g+1}) < \varepsilon \]

where,
\[ f(\vec{a}^{g+1}) = \| \vec{x} - \vec{x}^{DE} \|^2 + \| \vec{z} - \vec{z}^{DE} \|^2 \]

or if the number of pre-selected generations reached, optimizer will stop, otherwise it will start again from step 2. Here, \( \vec{x}^{DE} \) and \( \vec{z}^{DE} \) are calculated by using \( \vec{a}^{g+1} \) in equations (1) and (2) respectively. The summary of proposed method is given in Table 1.

### 5. SIMULATION RESULTS

The performance of proposed array geometry with proposed optimization algorithm is verified in comparison with PSO and GA by simulation experiments. A Intel(R) Core(TM) i5-4590 CPU @3.30GHz and 8G RAM is used. The algorithms are implemented without special code optimization or hardware acceleration in MATLAB R2015a. The sensors in each array are separated by equal distances of half-wavelength. The ranges for elevation and azimuth angles are taken [0°, 180°]. Different cases are discussed and for all cases, the results are averaged over two hundred independent runs. Noise is taken as white Gaussian for all cases. For DE, number of generations (NOG) = 500, Cross over rate (CR) = 0.5, number of chromosomes (NOC) = 50 and the constant \( F \) is set to 0.5 for PSO, swarm size is set to 110 and population size for GA is taken 500 for all cases.

**Case 1:** RMSE versus SNR — The performance of proposed method w.r.t. SNR is investigated in this case. Two near field sources from \((r = 25\text{m}; \Theta = 30^\circ; \varphi = 50^\circ)\) and \((r = 90\text{m}; \Theta = 120^\circ; \varphi = 150^\circ)\) are considered. SNR varies from -10 dB to 15 dB with an interval of 5 dB. Fig. 2 and Fig. 3 shows the RMSEs of angle estimations and range estimations respectively. It can be observed from Fig. 2 and Fig. 3 that even at very low values of SNR, DE outperforms the other techniques for joint estimation of range and 2-D DOA of near field sources. As the SNR is increasing, the estimation performance of DE becomes more accurate.

**Case 2:** RMSE versus \((M,N)\) -- The performance of proposed method w.r.t. number of array sensors used \((M,N)\) is investigated in this case. Two near field sources from \((r = 25\text{m}; \Theta = 30^\circ; \varphi = 50^\circ)\) and \((r = 90\text{m}; \Theta = 120^\circ; \varphi = 150^\circ)\) are considered. SNR is fixed at 10 dB whereas \((M,N)\) varies from

<table>
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<th>Table 1. Summary of 2-D near field source localization method</th>
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| **Given:** Received signals ‘\( \hat{x} \)’ and ‘\( \hat{z} \)’ from eq. 3 and eq. 4 along with pre-estimated number of sources ‘\( K \)’.
| **Step 1:** Initially create the present generation of chromosomes containing genes as follows:
\[ \vec{a}^g = [r_1 \cdots r_k \ \theta_1 \cdots \theta_k \ \varphi_1 \cdots \varphi_k]^T \]
| **Step 2:** If the fitness of any chromosome in present generation is up to the optimization criteria, then optimizer stops and outputs the estimated required parameters, otherwise it creates new chromosomes for next generation.
\[ \vec{a}^{g+1} = [r_1 \cdots r_k \ \theta_1 \cdots \theta_k \ \varphi_1 \cdots \varphi_k]^T \]
| **Step 3:** If the fitness of any chromosome in new generation is up to the optimization criteria, then optimizer stops and outputs the estimated required parameters, otherwise it will go back to step 2.

13
Fig. 2: RMSE of $\theta, \phi$ vs SNR

Fig. 3: RMSE of range vs SNR

Fig. 4: RMSE of $\theta, \phi$ vs $(M,N)$

Fig. 5: RMSE of range vs $(M,N)$

6 to 12 with an increment of 2 sensors. Fig. 4 and Fig. 5 shows the RMSEs of angle estimations and range estimations respectively. It can be observed from Fig. 4 and Fig. 5 that with the increase of number of array sensors used, the estimation performance of all algorithms become better but DE leave behind all other techniques even when less number of sensors are used for joint estimation of range and 2-D DOA of near field sources.

Case 3: RMSE versus $K$ -- The performance of proposed method w.r.t. number of sources ‘$K$’ impinging on the L-type array is investigated in this case. SNR and number of array elements are fixed at 10 dB and $M = N = 8$ respectively. One, two and three near field sources from $(r = 25m; \Theta = 20^\circ; \varphi = 40^\circ)$, $(r = 25m; \Theta = 20^\circ; \varphi = 40^\circ)$ and $(r = 50m; \Theta = 60^\circ; \varphi = 80^\circ)$, and $(r = 75m; \Theta = 100^\circ; \varphi = 120^\circ)$ are considered. Fig. 6 and Fig. 7 shows the RMSEs of angle estimations and range estimation respectively. It can be observed from Fig. 6 and Fig. 7 that with the increase of number of sources impinging on the L-type array, the estimation performance is degraded for all algorithms as the complexity has been increased but even then DE outperforms other techniques.

Case 4: Runtime versus $(M,N)$ -- The complexity analysis of proposed method as a function of runtime w.r.t. number of array sensors used $(M,N)$ is done in this case. Two near field sources from $(r = 25m; \Theta = 20^\circ; \varphi = 40^\circ)$ and $(r = 90m; \Theta = 120^\circ; \varphi = 150^\circ)$ are considered. SNR is fixed at 10 dB whereas $(M=N)$ varies from 6 to 12 with an increment of 2 sensors. It
can be observed from Fig. 8 that with the increase of number of array sensors used, the runtime for joint estimation of range and 2-D DOA of near field sources is increased as the complexity is increased at the cost of improved accuracy. However, DE still takes the least time to converge as compared to other techniques.

6. CONCLUSION

A novel, fast, low complexity method for joint estimation of range and 2-D DOAs (elevation and azimuth) of near field sources is presented in this paper which does not require any pair matching for the estimated parameters. The proposed method takes the advantage of maximum likelihood principle and exploits mean square error as fitness evaluation function to lower the mathematical complexity as it only requires a single snapshot of array output to converge and give optimized estimation. An evolutionary global optimizer named Differential Evolution is used as it does not get stuck in local minima and it is mathematically easy and fast to implement. Simulation results have verified the efficacy of proposed method in comparison with other methods.

The scope of this work in future is to enhance this technique for joint estimation of range and 2-D DOAs (elevation and azimuth) of near field wideband sources using microphone array without requiring any pair matching for the estimated parameters and/or to introduce array perturbations and check the effects of array perturbations on the performance of the proposed model.

7. ACKNOWLEDGMENTS

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8. REFERENCES


